

The Plasma Puddle as a Perturbative Black Hole

Clifford Cheung and Jared Kaplan

Jefferson Physical Laboratory, Harvard University, Cambridge, MA 02138

E-mail: cwcheung@fas.harvard.edu, kaplan@physics.harvard.edu

ABSTRACT: We argue that the weak coupling regime of a large N gauge theory in the Higgs phase contains black hole-like objects. These so-called “plasma puddles” are meta-stable lumps of hot plasma lying in locally un-Higgsed regions of space. They decay via $\mathcal{O}(1/N)$ thermal radiation and, perhaps surprisingly, absorb all incident matter. We show that an incident particle of energy E striking the plasma puddle will shower into an enormous number of decay products whose multiplicity grows linearly with E , and whose average energy is independent of E . Once these ultra-soft particles reach the interior they are thermalized by the plasma within, and so the object appears “black.” We determine some gross properties like the size and temperature of the the plasma puddle in terms of fundamental parameters in the gauge theory. Interestingly, demanding that the plasma puddle emit thermal Hawking radiation implies that the object is black (i.e. absorbs all incident particles), which implies classical stability, which implies satisfaction of the Bekenstein entropy bound. Because of the AdS/CFT duality and the many similarities between plasma puddles and black holes, we conjecture that black objects are a robust feature of quantum gravity.

1. Introduction

The AdS/CFT correspondence [1], [2] [3] has greatly improved our understanding of both gravity and gauge theory by providing a concrete realization of the holographic principle. For example, much work has been devoted to studying strongly coupled quasi-CFT dynamics using perturbative gravity. Conversely, CFTs have been useful for illuminating aspects of black hole physics, including the unitarity of Hawking evaporation.

It has been argued [4] that there exist black holes that can be localized in the IR of asymptotically AdS geometries, and that these solutions are dual to “plasma balls” in a confining CFT (for related work, see [5], [6], [7]). These plasma balls are meta-stable lumps of hot gluon plasma, and like black holes, they absorb all incoming matter and radiate thermally. Interestingly, since the AdS/CFT duality maps quantum effects to classical effects and vice versa, Hawking radiation is nontrivial from the gravitational point of view but straightforward in terms of the dual plasma ball description. Conversely, the “blackness” of black holes – their ability to absorb all incoming particles – is not obvious in the confining CFT picture.

In particular, consider the CFT dual of a particle thrown into a black hole: a glue ball thrown into a plasma ball. Naively, it seems that with sufficiently high energy such a glue ball would blast through, in the same way that an extremely high energy proton might barrel through the RHIC fireball. From this point of view, the most fundamental property of black holes – that they absorb all incoming particles – appears to be violated.

In [4], this problem was beautifully solved by taking into account the parton substructure of the incident particles [8], in accordance with Susskind’s ideas [9, 10]. In the dual gauge theory, the only available objects outside the plasma ball are mesons and glue balls, and at large ’t Hooft coupling these highly boosted hadrons contain a huge number of soft partons. Thus it is simply impossible to fire a high energy parton into the plasma ball – instead, an incoming glue ball fragments into many low energy partons which are promptly absorbed.

The purpose of the present paper is to explore the possibility that a *weakly* coupled gauge theory might furnish a perturbative dual to a black hole. At first this might seem like an unlikely prospect, particularly since the dual of a weakly coupled CFT is a strongly coupled gravitational theory. Indeed, it is not unreasonable to expect a phase transition between the weak and strong coupling regimes of the gravitational theory, corresponding to immensely different CFT physics in the two regimes. Despite these expectations, however, we will argue that “plasma puddles” in a weakly coupled gauge theory are qualitatively very similar to plasma balls/black holes.

Our setup is similar to [4], except that we consider a perturbative gauge theory in the Higgs phase, rather than a strongly coupled gauge theory in the confining phase. The low energy theory is comprised of photons and W bosons, rather than mesons and glue balls. Specifically, we will study an $\mathcal{N} = 4$ $SU(N)$ SYM theory Higgsed down to $U(1)^{N-1}$. By heating up a region of space we can locally un-Higgs the gauge group (see figure 1), creating a spatially varying Higgs vev. This means that W bosons have a position dependent mass that will act as an effective potential for the enclosed plasma. We will see

that for sufficiently large puddles, the plasma has a temperature much less than the height of the enclosing potential, and so it is kinematically trapped. Thus, classical stability is ensured. That said, the plasma puddle does emit radiation in a thermal spectrum, but we find that its lifetime is

$$\tau \sim NR, \quad (1.1)$$

where R is the radius of the puddle, so plasma puddles are infinitely long lived in the large N (classical) limit.

Since our theory is weakly coupled, it is possible to analyze the absorptive properties of a plasma puddle using standard perturbation theory. Unlike glue balls, photons and W bosons have no partonic substructure – since they are elementary point particles, arbitrarily large boosts involve no partonic subtleties. Thus it would seem that at sufficiently high energies, a plasma puddle can be penetrated. However, incident particles actually experience something very similar to parton showering – they decay near the boundary of the plasma puddle, bifurcating into a large number of ultra-soft daughter particles whose ensemble energy is smeared over the surface of the puddle and eventually thermalized. Qualitatively, this matches the process of gravitational “hair removal” in which an infalling particle is delocalized over the surface of a black hole as it is absorbed. In fact, this showering can only occur at the plasma puddle boundary (the effect requires momentum nonconservation), which dovetails nicely with the notion that black hole absorption is a local effect at the event horizon, independent of the interior.

In order for a plasma puddle to absorb even the highest energy particles, an incident particle of energy E has to shower into a large number of decay products that are too soft to escape the enclosing potential. In particular, it is necessary that the average energy of the final decay products, E_{avg} , does not increase with increasing E . If this is not the case, then an arbitrarily large E implies a commensurately large E_{avg} , allowing the decay products to blast through the plasma unharmed. Thus, total absorption is only possible if the rate¹ of showering increases at least linearly with energy. For E less than a particular (large) threshold we show that this is the case: the rate of decay of a gauge boson to two W ’s is

$$\Gamma_{A \rightarrow WW} \sim \lambda E, \quad (1.2)$$

where λ is the ’t Hooft coupling. Moreover, even though the decay rate goes to a constant for incident particles above this threshold energy, we find that they are still absorbed. In particular, these high energy particles shower promptly inside the plasma puddle into a large number of decay products at the threshold energy, which in turn decay at a rate λE . Thus, a high energy incident particle will shower into $\mathcal{O}(\lambda ER)$ decay products each of energy $E_{\text{avg}} \sim (\lambda R)^{-1}$. We confirm this reasoning with a more precise argument in section 3.2. As long as E_{avg} is less than the height of the enclosing potential, these decay products are trapped inside the plasma puddle, and so the puddle appears black.

¹For a detailed calculation of the dimensionless *probability* of showering, see appendix B

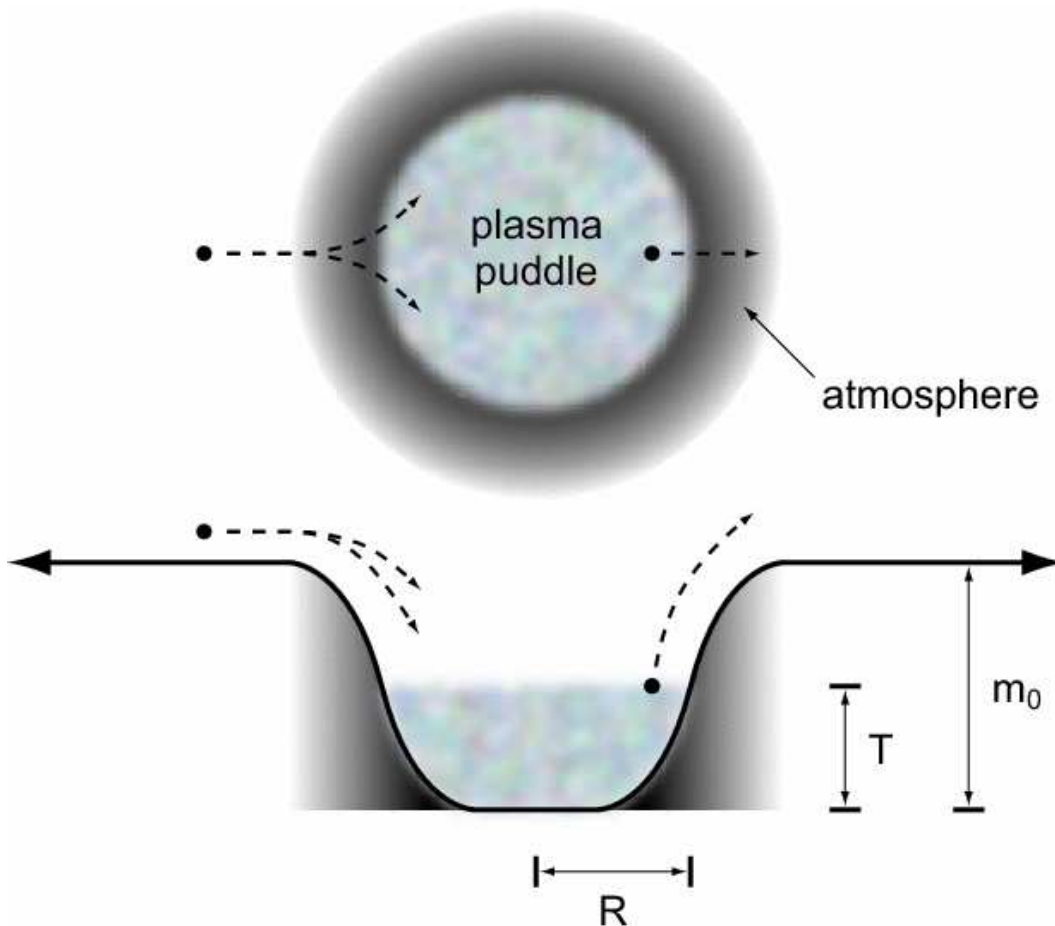


Figure 1: A plasma puddle cross section (above) and energy profile (below). The latter depicts the W mass as it asymptotes to a value of m_0 at infinity and vanishes within an un-Higgsed region of radius R . The plasma within settles into a hot puddle of temperature T . Both diagrams show a high energy incident particle on the left showering in the atmosphere, as well as some Hawking radiation escaping on the right.

Interestingly, as we show in section 4, if we demand that the plasma puddle emit thermal Hawking radiation, then it *automatically* also absorbs all incident matter, is classically stable, and satisfies the Bekenstein entropy bound [11]. During the process of Hawking evaporation, a stable plasma puddle will lose each of these properties sequentially, until it eventually becomes a free gas of gauge bosons. Since the plasma puddle is so similar to a black hole, our hope is that the large λ plasma ball/black hole duality established in [4] interpolates at small λ to a correspondence between the plasma puddle and some black object of a strongly coupled gravitational theory.

Note that for our purposes we will focus entirely on the gauge degrees of freedom of the $\mathcal{N} = 4$ SYM, ignoring gaugino partners and scalar moduli. Moreover, as we are interested only in parametric scalings, we will be largely ignoring numerical factors. The outline of the paper is as follows. In section 2 we determine the general properties of our

setup: gross properties of the plasma puddle, Hawking radiation, and classical stability. In section 3, we give a nice physical estimate of the rate of particle showering in the plasma puddle atmosphere. In section 4 we show that demanding thermal Hawking radiation immediately implies other black hole-like properties, and we conclude in section 5. The appendices contain a more formal derivation of the probability of decay.

2. The Plasma Puddle

In this section we give a detailed account of what a plasma puddle is and how it forms. Our setup is as follows. Consider an $\mathcal{N} = 4$ $SU(N)$ SYM at large N and weak 't Hooft coupling λ that is Higgsed down to $U(1)^{N-1}$ (note that due to the Higgsing, the theory is not conformal and so particles and S-matrices are well defined). The spectrum of the theory consists of $N - 1$ massless photons and $N^2 - N$ massive W bosons. Now, let us fire an ensemble of high energy W 's into a small region of space². The influx of W 's heats up the region and locally un-Higgses the gauge group. Once the W 's thermalize, the resulting meta-stable object is a plasma puddle.

The local un-Higgsing can be parameterized by a spatially varying Higgs vev which induces a spatially varying mass, $m(x)$, for the W bosons. The W mass profile vanishes inside the plasma puddle but asymptotes to some nonzero mass m_0 outside (the precise mass of individual W s depends on the Higgsing pattern, but the details will not be important). Let us define the atmosphere to be the region near the plasma puddle boundary in which the mass profile varies. Again, we emphasize that the W 's are confined by the $m(x)$ potential, while the photons are actually massless everywhere.

If we ignore all interactions, it is straightforward to see what happens to free streaming W 's as they collapse into a plasma puddle. Those W 's with $E > m_0$ escape to infinity while those with $E < m_0$ settle at the basin of the $m(x)$ potential. Due to the potential barrier, this puddle of W 's can never escape.

The story is similar if we include gauge interactions, except that the puddle of W 's thermalizes into a puddle of photons and W bosons. To see this, we compute the mean free path d of a gauge boson A_{ij} traversing the hot plasma. From color conservation, A_{ij} can only scatter off of some A_{jk} , leaving A_{il} and A_{lk} in the final state, where k and l are free indices. Summing the amplitude squared, g^4 , over phase space, k and l contribute N^2 to the cross section, yielding

$$\sigma \sim \lambda^2 T^{-2}, \quad (2.1)$$

since the temperature sets the energy scale of the interaction. The number density n of a gauge boson with a given pair of color indices is T^3 , so the mean free path is

$$d \sim (n\sigma)^{-1} \sim (\lambda^2 T)^{-1}. \quad (2.2)$$

²Note that an ensemble of W bosons that are initially at rest will naturally collapse due to the attractive force of dilaton gravity, as in described in [12]. In the nonrelativistic limit, this behaves exactly like true gravitational collapse.

For sufficiently large 't Hooft coupling the mean free path is smaller than the size of the plasma puddle and thus gauge bosons cannot go very far without scattering (see figure 2). Let us denote this as the highly thermalized regime. In this case the plasma puddle quickly thermalizes into a hot, homogenous soup of photons and W bosons at a temperature T . The plasma puddle is classically stable as long as its temperature is less than m_0 , so that the interior plasma is kinematically trapped. Without this condition, nothing prevents the plasma from simply escaping to infinity, and so the relation

$$T < m_0, \tag{2.3}$$

implies classical stability of the plasma puddle. In addition, since the plasma puddle is highly thermalized, only particles at the very surface have any hope of escape. We argue in section 2.2 that this phenomenon is an $\mathcal{O}(1/N)$ area effect that is thermal and dual to Hawking radiation. For these reasons (along with their absorptive properties, which we discuss in section 3) we claim that plasma puddles in the $d < R$ regime are black hole-like objects.

On the other hand, if λ is sufficiently small, then $d > R$ and a typical gauge boson can traverse the extent of the plasma puddle without ever scattering (see figure 2). However, given a time of order d , the plasma will eventually thermalize, yielding a collection of nearly free photons and W 's at a temperature T . The W bosons travel in straight lines through the puddle until they reach the potential barrier from $m(x)$, after which they roll back towards the interior, and repeat. As a result the W 's form a relativistic plasma at the bottom of the $m(x)$ potential, but they become nonrelativistic in near the $m(x)$ barrier wall, simply because they have less kinetic energy there. In contrast, the photons are massless everywhere and can free stream outwards. As we discuss in section 2.2, since plasma puddles in the $d > R$ regime do not Hawking radiate in the traditional sense, we do not identify them as dual black holes.

2.1 Gross Properties and Relations

Thus far our discussion has included T and R as a priori attributes of the plasma puddle. However, as we show in this section, these two variables are fixed in terms of the “universal” quantities N , λ , m_0 , and the total energy of the plasma puddle, M .³

To begin we note that unlike Schwarzschild black holes, plasma balls/puddles do not obey the relation $T \sim 1/R$ for the following reason. Since the plasma puddle has an entropy $S = N^2 T^3 R^3$, the relation $T \sim 1/R$ would imply that the entropy is independent of the size of the object. However, this is not the case – the entropy of a plasma puddle/ball increases with size. In the case of the strongly coupled plasma balls of [4], the temperature of large plasma balls is set by the confinement scale. Analogously, one might expect a similar situation for plasma puddles, i.e. that the temperature is given by the Higgs vev. We will see that this is not the case.

Once formed, a plasma puddle is an isolated system, so its total energy is conserved. However, its entropy should be maximized subject to this constraint, so we treat the plasma

³Here we use the symbol M in anticipation of matching this total energy of the plasma puddle to the mass of a dual black hole.

puddle with the micro-canonical ensemble. In $\mathcal{N} = 4$ SYM, the scalar fields Φ_{ij} are in the adjoint representation of the gauge group. When these fields Higgs $SU(N)$ to $U(1)^{N-1}$, their vevs can be written as diagonal $N \times N$ matrices with spatially varying components. The total energy of a plasma puddle arises from two contributions: the gradient kinetic energy from the (spatially varying) Higgs mechanism, and the thermal energy of the plasma within. Because these contributions are boundary and volume effects, respectively, we are essentially balancing the pressure against the surface tension. Neglecting $\mathcal{O}(1)$ factors, the total energy of the plasma puddle is thus

$$M = \int d^3x \text{Tr}(\nabla\Phi)^2 + N^2 T^4. \quad (2.4)$$

The second term only accounts for the relativistic ($T \gg m(x)$) region of the plasma. The nonrelativistic ($T \ll m(x)$) region has an energy density $N^2 T^2 m(x)^2 \exp(-m(x)/T)$, and this is always less than or equal to $N^2 T^4$. Thus neglecting this contribution merely amounts to a rescaling of the plasma energy by an $\mathcal{O}(1)$ factor.

Each diagonal entry of Φ gets a vev that asymptotes to $\sim m_0/g$ at infinity but vanishes in some region of size R . Moreover, let us define L to be the thickness of the atmosphere, i.e. the region in which the Higgs vev varies. Estimating the energy we find that

$$M = N^2 \left(\frac{m_0^2 (R+L)^2}{\lambda L} + T^4 R^3 \right). \quad (2.5)$$

Before maximizing the entropy at fixed energy, let us motivate why this fixes the radius of the plasma puddle. Consider what happens as we increase the radius of the puddle at fixed m_0 . This increases the gradient Φ energy, but since the total energy is fixed, this forces the temperature to diminish. Since the entropy goes as $T^3 R^3$, the increase in radius and decrease in temperature are competing effects. Hence, there is an extremal value for R such that the entropy is maximized. Note that this is what fixes the size of a balloon full of gas.

Fixing M allows us to solve for T , yielding an entropy

$$S = N^2 T^3 R^3 \quad (2.6)$$

$$= \sqrt{N} R^{3/4} \left(M - \frac{N^2 m_0^2 (R+L)^2}{\lambda L} \right)^{3/4}. \quad (2.7)$$

The entropy is maximized for $L \sim R$, because this maximizes the term in large parentheses. Maximizing the entropy with respect to R , we find that

$$R \sim \frac{\lambda M}{N^2 m_0^2}, \quad (2.8)$$

and also that

$$T \sim \frac{N m_0^{3/2}}{\lambda^{3/4} M^{1/2}}. \quad (2.9)$$

Thus we see that while strongly coupled plasma balls obey the relation $R \sim M^{1/3}$ [4], perturbative plasma puddles have that $R \sim M$.

Finally, let us add some remarks about the precise shape of the Higgs vev, Φ . Near the boundary of the plasma puddle it is nontrivial to determine Φ , because the plasma back-reacts on the Higgs vev and vice versa. However, outside R the Φ field is free, so it obeys Laplace's equation, and thus

$$\Phi \sim \left(1 - \frac{R}{r}\right) \quad \text{for } r > R, \quad (2.10)$$

obtains. Thus the scalar field atmosphere has a $1/r$ tail, so again we see that the thickness L of the plasma puddle is of order R . In $d \geq 4$ spacetime dimensions (in the gauge theory), the Higgs profile would go as

$$\Phi \sim 1 - \left(\frac{R}{r}\right)^{d-3}, \quad (2.11)$$

so in higher dimensions there are no IR subtleties. For this reason we do not expect that this tail is qualitatively important for black hole/plasma puddle physics.

2.2 Hawking Radiation

In the gravitational picture, the total absorption of incident particles by a black hole is obvious at the classical level, while Hawking radiation is a nontrivial quantum effect. In contrast, in our CFT setup we will see that plasma puddle absorption is nontrivial but Hawking radiation is manifest. In this section we consider the latter.

To begin, let us consider the regime in which $d < R$ and the enclosed plasma is highly thermalized. In this case the gauge bosons cannot go very far without scattering, and thus a particle has no hope of escape unless it is at the very surface of the plasma puddle. Moreover, since the $m(x)$ potential kinematically bounds W bosons, a surface particle can only escape if it is a photon. Since only $1/N$ of the particles is a photon (due to the Higgsing pattern $SU(N) \rightarrow U(1)^{N-1}$), the plasma puddle radiates photons in a thermal spectrum at temperature T . Since this is a $\mathcal{O}(1/N)$ effect, it is natural to associate this radiation to $\mathcal{O}(\hbar)$ Hawking radiation in a gravitational dual.

Next, let us compute the rate at which a plasma puddle loses energy as the result of Hawking radiation. In an infinitesimal time interval dt , an order one fraction of photons that are within dt of the surface of the plasma puddle will exit. If n_γ is the number density of photons in the plasma, then there are $n_\gamma R^2 dt$ photons in this region near the surface. Since each has an energy of order T ,

$$\frac{dE}{dt} = -n_\gamma R^2 T. \quad (2.12)$$

Assuming that these photons are in thermal equilibrium, we find

$$\begin{aligned} \frac{dE}{dt} &= -N R^2 T^4 \\ &= -\frac{N m_0^2}{\lambda}, \end{aligned} \quad (2.13)$$

so we see that at fixed temperature the energy loss is an area effect, while at fixed m_0 the energy loss is completely independent of size and temperature. The lifetime is given by the time it takes for the total energy M to be radiated

$$\tau = -M \left(\frac{dE}{dt} \right)^{-1} \sim NR, \quad (2.14)$$

so plasma puddles are completely stable in the classical limit $N \rightarrow \infty$.

The situation is more complicated if $d > R$, so let us be more careful. We consider scattering inside the plasma puddle for arbitrary d and R . If the overall number density of particles is n_{tot} , then there is a total of $n_{\text{tot}}R^3$ particles, and so the total rate of scattering processes is $n_{\text{tot}}R^3/d$. Of these processes, an $\mathcal{O}(1/N)$ fraction create additional photons. Likewise, there are $n_\gamma R^3$ photons in the plasma, so the rate for scattering events involving photons is $n_\gamma R^3/d$. Almost all of these scattering events destroy photons, because $AW \rightarrow AW$ is suppressed by a factor N compared to $AW \rightarrow WW$. Thus scattering events change the number of photons in the plasma at a rate

$$\left. \frac{d(n_\gamma R^3)}{dt} \right|_{\text{scattering}} = \left(\frac{1}{N} n_{\text{tot}} - n_\gamma \right) R^3 \frac{1}{d}. \quad (2.15)$$

However, radiation leaving the plasma puddle also decreases the number of photons at the rate

$$\left. \frac{d(n_\gamma R^3)}{dt} \right|_{\text{radiation}} = -n_\gamma R^2. \quad (2.16)$$

These two processes balance when

$$n_\gamma \sim \frac{1}{N} n_{\text{tot}} \left(1 + \frac{d}{R} \right)^{-1}. \quad (2.17)$$

Thus we see that as d becomes larger than R , the number of photons becomes less than $1/N$ of the total number of particles, and so we leave thermal equilibrium. In particular, once $d > R$, the rate of Hawking radiation begins to decrease by the significant factor R/d . Thus, in the $d > R$ regime the plasma puddle does not radiate a thermal spectrum, and exiting photons can free stream from anywhere within the plasma, not just the surface. For this reason we do not consider such a plasma puddle to be the dual of a black hole.

We have glossed over a subtlety in our treatment of Hawking radiation. As we argue in section 3, a high energy photon striking the plasma puddle atmosphere will shower into many other gauge bosons, until all of the decay products have a tiny energy of order $(\lambda R)^{-1}$. But since it is possible that $T > (\lambda R)^{-1}$ (in fact, an even stronger condition is required to be in the highly thermalized regime), we might worry that an outgoing photon will simply shower in the plasma puddle atmosphere. If this happens, its low energy decay products are likely to be reabsorbed by the plasma puddle, leaving no reason to expect Hawking radiation, let alone a thermal distribution of Hawking radiation.

The resolution to this puzzle is that once an outgoing photon makes it to the outer atmosphere of the plasma, it does not have the $2m(x)$ of energy necessary to shower into

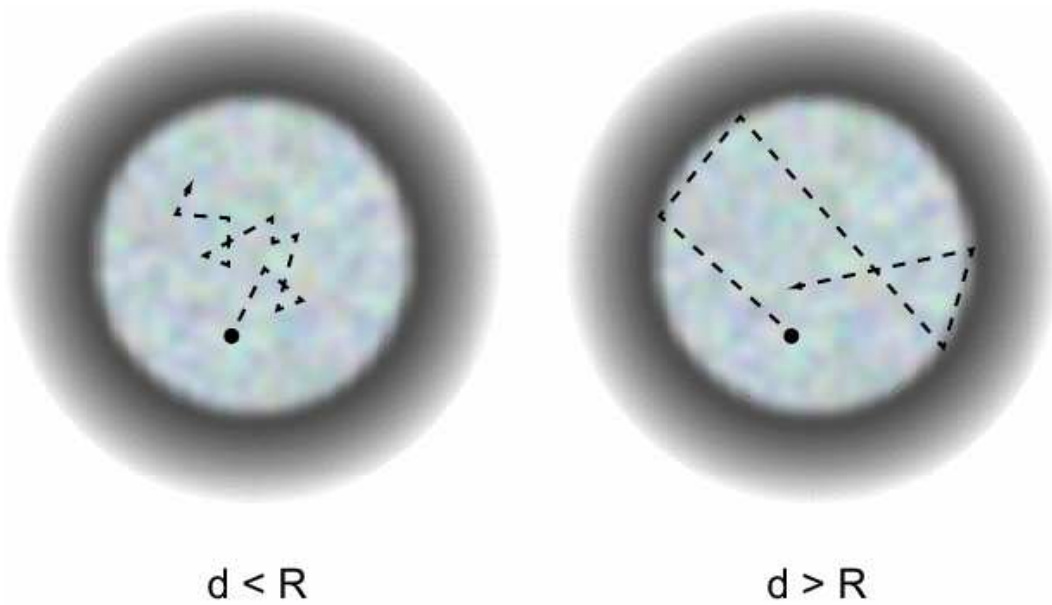


Figure 2: Here we show the typical trajectory of a gauge boson in the highly thermalized regime, $d < R$, and otherwise. Hawking radiation is only thermal when $d < R$.

two W bosons, since it only has energy T . Thus, kinematical constraints allow photons at temperature T to escape from the plasma puddle as thermal Hawking radiation, even though high energy incident photons typically shower as they fall in.

Finally, let us discuss briefly the possibility of W boson Hawking radiation. While most of the W bosons are of course kinematically trapped, there is a Boltzmann tail in the thermal distribution which allows an $e^{-m(x)/T}$ fraction of the W 's to escape. While this effect is exponentially suppressed, it is not suppressed by powers of $1/N$. This is puzzling from the standpoint of the gravity dual, in which all Hawking radiation should be an $\mathcal{O}(\hbar)$ effect. Understanding the true N dependence of W boson Hawking radiation will require further study.

3. Plasma Puddle Absorption

The primary feature of the plasma puddle that distinguishes it from more conventional objects that are hot and stable (such as stars) is its ability to absorb all incident matter. In this section we verify that the plasma puddle is black by showing that a high energy incident particle will shower into numerous soft decay products that thermalize with the interior plasma. While a formal calculation of the decay probability is given in appendix A, we present a more direct physical argument in the following section.

3.1 Estimation of the Showering Rate

To begin, let us consider the propagation of a single photon through free space. The photon is absolutely stable due to kinematics and phase space – momentum conservation

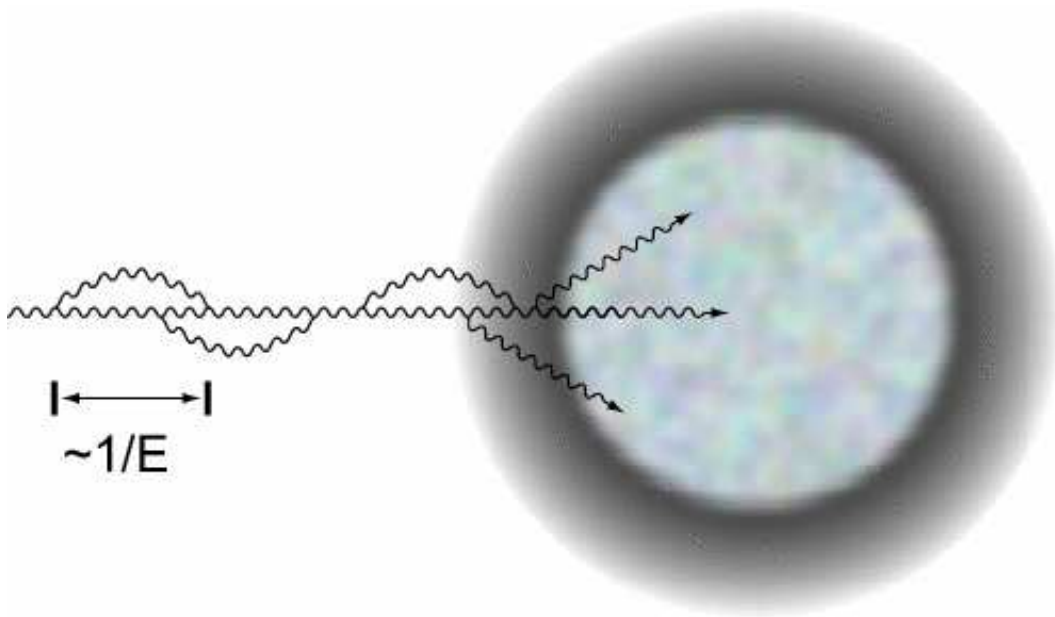


Figure 3: An incoming gauge boson is surrounded by a cloud of virtual particles. Only when the gauge boson strikes the plasma puddle atmosphere can these virtual particles become real. The characteristic length scale of quantum fluctuations is given by $(\lambda E)^{-1}$.

only allows decays into exactly collinear, massless decay products and since this is a set of measure zero in phase space the decay rate vanishes. However, in the presence of momentum violation (such as the plasma puddle atmosphere), noncollinearities are allowed and so the decay rate is nonzero.

To estimate this rate, we invoke the uncertainty principle, which tells us that a photon in the vacuum is immersed in a $\mathcal{O}(\lambda)$ cloud of virtual gauge bosons constantly coming in and out of existence. Since the only relevant time scale for a massless particle in free space is its energy E , the photon produces virtual particles of energy $\sim E$ at a rate $\Gamma \sim \lambda E$. Indeed, once the photon strikes the plasma puddle atmosphere, the resulting momentum violation allows for these virtual particles to become real⁴. Since Γ is the rate at which photons are produced, and in the momentum violating background they can simply escape to infinity, we expect that

$$\Gamma_{A \rightarrow WW} \sim \lambda E. \quad (3.1)$$

The reader may suspect that this answer breaks down at large energy, and this is in fact the case. There are several reasons why we might expect this to happen. First of all, if the photon is arbitrarily boosted (relative to the rest frame of the plasma puddle), it eventually reaches the threshold energy for plasma puddle creation, and so the correct

⁴This is a common effect, and is detailed in standard textbooks on quantum field theory [14]. For example, consider the electron, which carries with it a number density of virtual photons given by $N_\gamma \sim \alpha$. Bremsstrahlung effects from the scattering of an electron off of a charged target can be understood as virtual photons becoming real due to a nontrivial background.

calculation would involve plasma puddle/plasma puddle scattering. This is akin to firing a particle into a black hole at trans-Planckian energies, which is really the same as black hole/black hole scattering.

Secondly, at very high energies the W decay products of the photon are so collinear that their splitting is completely unmeasurable (by the plasma puddle) and is thus unphysical. Let us illustrate this soft collinear divergence more explicitly. As noted previously, the photon cannot decay into two exactly collinear W bosons due to phase space, but if it receives of a momentum “kick”, then it can decay into W s which split in the transverse direction. However, it can only decay in this way in the momentum violating background. In the plasma puddle atmosphere, where the plasma itself is negligible, the only source of momentum violation (position dependence) is the gradient of the Higgs field, so we expect that the kick is proportional to $1/R$. The angle subtended by the outgoing W ’s is approximately k_{\perp}/E , where $k_{\perp}^2 \sim E/R$ is the transverse momentum difference between the W ’s⁵. Demanding that the “cone” traced out by the outgoing particles grows to a size greater than their Compton wavelength $\sim 1/m_0$, we find that

$$R \frac{k_{\perp}}{E} > \frac{1}{m_0} \quad \Rightarrow \quad E < m_0^2 R. \quad (\text{soft collinear bound}) \quad (3.2)$$

If this bound is violated, then there is a soft collinear divergence and the splitting is unmeasurable. Note that if the energies of the decay products are different, only the lesser of the two need satisfy this bound.

In appendix A.4 we address these high energy issues directly by calculating the rate of decay across the extent of the plasma puddle, yielding

$$\Gamma_{A \rightarrow WW} \sim \lambda E, \quad (E < m_0^2 R) \quad (3.3)$$

$$\sim \lambda m_0^2 R, \quad (E > m_0^2 R, \text{Bremsstrahlung}) \quad (3.4)$$

$$\sim \frac{\lambda m_0^4 R^2}{E}, \quad (E > m_0^2 R, \text{symmetric decay}), \quad (3.5)$$

where “Bremsstrahlung” denotes the region in phase space in which one of the outgoing particles is much softer than the other (that is, with energies of order $m_0^2 R$ and $E - m_0^2 R$) and “symmetric decay” denotes the regime in which they are comparably energetic (with energies of order E). Since the probability of a symmetric decay is suppressed by a factor of $m_0^2 R/E$ relative to Bremsstrahlung, soft emission totally dominates showering at energies above the threshold $m_0^2 R$. For example, consider an incoming particle boosted to some high energy a hundred times greater than $m_0^2 R$. Emission of a soft particle of energy $m_0^2 R$ is a hundred times more likely than decay into two particles of comparable energy. Moreover, once this Bremsstrahlung occurs, the harder particle only loses $m_0^2 R$ energy, and so a consecutive soft emission is still ninety nine times more likely than a symmetric decay, and so on. Before long, the particle emits enough Bremsstrahlung that its energy drops to $m_0^2 R$ and its decay rate begins to scale linearly with energy. In the following section we consider the physical consequences of these rates.

⁵We estimate k_{\perp} as follows. We begin with an incident photon with 4-momentum $(E, 0, 0, E)$ and assume that it receives a momentum “kick” $(0, 0, 0, -1/R)$. This allows it to split into two massless particles with $k_{\perp}^2 \approx E^2 - (E - 1/R)^2 \sim E/R$

3.2 Absorption and E_{avg}

In section 3 we saw that $P_{A \rightarrow WW}$ increases linearly with E up to the scale $m_0^2 R$, after which it remains constant. Next, we show that this implies that an incident particle, no matter how energetic, will always be absorbed by the plasma puddle.

To begin, let us consider showering in the $E < m_0^2 R$ regime. An incident particle of energy E will decay in the plasma puddle atmosphere at a rate λE . In turn, its decay products have a smaller rate for decaying simply because they have less energy. In fact, after a long sequence of decays the average energy of the final decay products eventually diminishes to $E_{\text{avg}} \sim (\lambda R)^{-1}$. At this point the decay rate dips below $1/R$, and the ultra-soft daughter particles can traverse the extent of the plasma puddle without showering. Thus, showering completely terminates once the decay products reach an energy of $(\lambda R)^{-1}$. Since the final decay products have a Compton wavelength proportional to the radius of the plasma puddle, showering effectively de-localizes the incident particle over the entire puddle. This picture matches nicely with Susskind's ideas about black hole absorption [9, 10].

In addition, as long as this terminal energy is less than m_0 , the daughter particles are kinematically trapped by the Higgs profile, and they will eventually be thermalized. This translates into a blackness bound

$$\frac{1}{\lambda R} < m_0, \quad (3.6)$$

which must be satisfied if the plasma puddle is to absorb all incident matter.

We now argue that the above conclusions are still valid even above the threshold energy, $m_0^2 R$. As we argued earlier, such a high energy particle favors the emission of soft particles of energy $m_0^2 R$. However, because the decay rate for Bremsstrahlung remains constant at high energies, one might wonder whether a sufficiently energetic particle might blast through the plasma puddle. A careful analysis shows this is not the case.

In particular, let us very roughly estimate the rate of $1 \rightarrow n$ showering for an incident particle of energy $E = nm_0^2 R$, where n is some larger integer. Noting that a $1 \rightarrow n$ decay includes a sequence of $i \rightarrow i + 1$ sub-processes, and is often dominated by on-shell regions of phase space, we can estimate the rate of $1 \rightarrow n$ decay by

$$\Gamma_{1 \rightarrow n} = \frac{1}{R} P_{1 \rightarrow 2} \times P_{2 \rightarrow 3} \times \dots \times P_{n-1 \rightarrow n}, \quad (3.7)$$

$$= \frac{1}{R} (\lambda m_0^2 R^2)^n, \quad (3.8)$$

where schematically $P \sim \Gamma R$ (see appendix A, where we calculate these probabilities explicitly). As we show in section 4, classical stability immediately implies that the quantity in parentheses is greater than unity. Thus, we argue that an incident particle of energy $nm_0^2 R$ will decay into n particles of energy $m_0^2 R$ within a region of size R . After this sequence of Bremsstrahlung events, the decay products all have energies of order $m_0^2 R$, and so we can apply our analysis for the $E < m_0^2 R$ regime to these decay products. Since these particles decay to a multitude of ultra-soft particles of energy $(\lambda R)^{-1}$, we find that total absorption occurs, no matter what the incident energy.

Interestingly, we are finding that large multiplicity events dominate at high energies, which is reminiscent of Hawking evaporation – after all, the probability that a large black hole will decay to two particles is extremely small, because it is suppressed by e^{-S} , while the decay to a huge number of soft particles is virtually guaranteed. Thus the necessity of including $1 \rightarrow n$ decays at high energies seems to indicate that high energy particles carry a great deal of entropy.

4. Implications of $d < R$

In section 2.1 we argued that in the highly thermalized regime ($d < R$) the plasma puddle emits thermal Hawking radiation at the surface. In this section we show that $d < R$ also implies total absorption, which implies classical stability, which implies satisfaction of the Bekenstein entropy bound.⁶ For this reason, we identify $d < R$ plasma puddles as black hole-like objects.

Applying our relations from section 2.1, we can rewrite the $d < R$ bound, the blackness bound (Eq. (3.6)), the classical stability bound (Eq. (2.3)), and the Bekenstein bound as

$$m_0 R > \lambda^{-7/2}, \quad (\text{highly thermalized}) \quad (4.1)$$

$$> \lambda^{-1}, \quad (\text{total absorption}) \quad (4.2)$$

$$> \lambda^{-1/2}, \quad (\text{classical stability}) \quad (4.3)$$

$$> \lambda^{1/2}, \quad (\text{Bekenstein bound}). \quad (4.4)$$

The above sequence of bounds gives an interesting depiction of what happens during plasma puddle evaporation. As the energy of the plasma puddle diminishes via Hawking radiation, its size shrinks commensurately. First, the plasma puddle leaves the highly thermalized regime and stops emitting thermal Hawking radiation. Next, the plasma puddle becomes transparent to incident matter, and finally the object becomes classically unstable. After this point, the enclosed photons and W bosons in the plasma simply stream outwards, leaving a free gas of gauge bosons. It would be interesting to explicitly connect these processes with the Horowitz-Polchinski transition [15], [16], [17].

It is also noteworthy that the Bekenstein bound is precisely equivalent to the condition

$$T > \frac{1}{R}, \quad (4.5)$$

which is necessary for the application of classical thermodynamics. As this bound is approached, the energy/size of the system begins to saturate the uncertainty principle, and we are forced to count individual quantum mechanical microstates.

5. Discussion

In this paper we have argued for the existence of black hole-like objects living in large N gauge theories at weak 't Hooft coupling. Since these theories are completely perturbative, we can calculate much of the physics. In particular, in the regime in which the

⁶The Bekenstein entropy bound gives an upper bound on the entropy of a system in terms of its gravitational mass M and its size R : $S < MR$.

mean free path d is smaller than R , we find that these meta-stable puddles of plasma are classically stable and emit radiation in a thermal spectrum. While these properties are of course common to any hot star-like object, we moreover find that the plasma puddle absorbs all incident matter, no matter how energetic. This occurs because high energy particles invariably shower into ultra-soft decay products that are kinematically bound by the effective potential from the spatially varying Higgs vev. All in all, we find this to be compelling evidence that the plasma puddle is dual to a black object in a strongly coupled gravitational theory.

In addition, our work gives a particularly nice picture of plasma puddle evaporation, which may be connected to the Horowitz-Polchinski transition [15], [16], [17]. Indeed from section 4 we saw that as the object radiates away energy, it eventually leaves the $d < R$ regime and stops emitting thermal Hawking radiation. After even more energy loss, the plasma puddle stops being black, and eventually becomes classically unstable. Interestingly, we also find that the $d < R$ bound gives a nice lower bound on the total energy of a plasma puddle, given by

$$M > \lambda^{-9/2} N^2 m_0. \quad (5.1)$$

Since a plasma puddle must have at least this much energy to form a black hole-like object, we might interpret the left-hand side as the CFT dual to the Planck mass.

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A. Detailed Absorption Computation

In this section we give a detailed derivation of the probability for the decay of a photon to two W bosons as it passes through the plasma puddle atmosphere. Although we are formally calculating a *probability*, we can re-interpret it as a decay rate via $\Gamma \sim P/R$. To begin, we consider a toy scalar model for simplicity. This scalar field theory has an action

$$S = \frac{1}{2} \int \partial\phi^2 + \partial\chi^2 - (m^2(x_3) + g\phi)\chi^2. \quad (A.1)$$

In this theory, a massless scalar field ϕ is coupled cubically to a scalar χ whose mass m^2 is a nontrivial function of x_3 . Here ϕ and χ are scalar analogs of the photon and the W boson, respectively, and m^2 mimics the effects of a space varying Higgs vev on the W mass. Notice that we have made the simplification that the plasma puddle is infinite in the x_1 and x_2 directions, which is a very good approximation if the plasma puddle is large and the incoming particles approach from the x_3 direction.

A.1 Propagator

Next, let us compute the χ propagator in the regime in which the incoming and outgoing energies are much larger than the characteristic energy scale $1/R$ set by the mass profile. To do this we use the WKB approximation to solve the wave equation⁷. The wave equation is given by

$$[\square + m^2] \chi = 0, \quad (\text{A.2})$$

where the space dependent mass is

$$m^2(x_3) = m_0^2(1 - B(x_3)), \quad (\text{A.3})$$

and $B(x_3)$ is a “bump” function which vanishes at infinity and peaks to unity in a compact region of size R . From section 2.1 we know that $B(r)$ goes as $1/r$ for large r , but for now let us take $B(x_3)$ to be a general function. Consider the following WKB ansatz solution:

$$\chi(x) = \exp(i\tilde{p}(x)x) \quad (\text{A.4})$$

$$\tilde{p}(x) \equiv \left(p_0, p_1, p_2, p_3 + \frac{m_0^2}{2p_3} f(x_3) \right), \quad (\text{A.5})$$

where $f(x_3)$ is a function that will be determined by plugging χ into the wave equation. From now on, a tilde on a momentum variable will represent a nontrivial space dependence of this kind.

If we plug this ansatz back into the wave equation, we find

$$[\square + m^2] \chi = \left[-p^2 + m_0^2 + m_0^2 \left(-B + f + x_3 f' - \frac{if'}{p_3} - \frac{ix_3 f''}{2p_3} \right) \right] \chi. \quad (\text{A.6})$$

If the quantity in curved parentheses vanishes, then χ is an eigenstate of the wave equation. In the regime where $p_3 \gg 1/R$, the terms containing $1/p_3$ are small, since they go as inverse powers of $p_3 R$, so we drop them. Now it is easy to solve for f in the resulting differential equation

$$-B + f + x_3 f' = 0, \quad (\text{A.7})$$

yielding

$$\tilde{p}(x) = \left(p_0, p_1, p_2, p_3 + \frac{m_0^2}{2p_3 x_3} \int^{x_3} B(x'_3) dx'_3 \right). \quad (\text{A.8})$$

Thus in the WKB approximation, the χ propagator is

$$G_\chi(x, y) = \int \frac{i}{p^2 - m_0^2} \times e^{i(\tilde{p}(x)x - \tilde{p}(y)y)}, \quad (\text{A.9})$$

to zeroth order in $1/(ER)$ and all orders in m_0^2 .

⁷If we include m_0 as a mass insertion in Feynman diagrams, then our answer is necessarily a series expansion in m_0 . Using the WKB approximation, we will be neglecting terms suppressed by higher powers of $1/(ER)$, but keeping terms to all orders in m_0 .

A.2 Amplitude

Next, let us consider the amplitude for a ϕ particle of momentum p to decay into two χ particles of momenta k and q in the spatially varying background. To do so we write down the corresponding time ordered 3-point correlator in coordinate space

$$\langle \chi(y)\chi(z)\phi(w) \rangle \sim g \int G_\chi(y-x)G_\chi(z-x)G_\phi(x-w)d^4x. \quad (\text{A.10})$$

Next, we Fourier transform to the variables p , k and q . In accordance with the LSZ reduction formula, a scattering amplitude corresponds to the Fourier transform of the appropriate time ordered correlator with the pole from each external leg stripped off. Amputating the legs, we find that the Feynman amplitude is

$$\mathcal{M}(k, q, p) = g \times \int d^4x e^{i(\tilde{k}(x)+\tilde{q}(x)-p)x}, \quad (\text{A.11})$$

$$= g \times \delta^{(012)}(k+q-p)F(k_3, q_3, p_3) \quad (\text{A.12})$$

$$F(k_3, q_3, p_3) = \int dx_3 e^{i(\tilde{k}_3(x_3)+\tilde{q}_3(x_3)-p_3)x_3}. \quad (\text{A.13})$$

where g is the dimensionful coupling strength. Notice that in the $m_0^2 \rightarrow 0$ limit, F reverts to a full 4-momentum conserving delta function, as expected. Next, let us evaluate F .

By separating a factor of $\exp i(k_3 + q_3 - p_3)x$ in the integrand of F , it is clear that F is simply the Fourier transform of the quantity

$$\exp \left[im_0^2 \left(\frac{1}{k_3} + \frac{1}{q_3} \right) \int^{x_3} B(x'_3) dx'_3 \right]. \quad (\text{A.14})$$

Let us consider the case in which the Higgs profile is a square bump function and so B is unity for $|x_3| < R$ and 0 otherwise. Given this simplification the integral is easy to evaluate piece-wise and F takes the simple form

$$F \sim \lim_{\epsilon \rightarrow 0} \frac{\sin(a/\epsilon + Rb)}{a} - \frac{b \sin R(a+b)}{a(a+b)}, \quad (\text{A.15})$$

$$\sim \pi \delta(a) - \frac{b \sin R(a+b)}{a(a+b)}, \quad (\text{A.16})$$

where we are using ϵ to regulate the δ function, and for convenience we have defined

$$a = k_3 + q_3 - p_3, \quad (\text{A.17})$$

$$b = m_0^2 \left(\frac{1}{k_3} + \frac{1}{q_3} \right). \quad (\text{A.18})$$

Since R is the largest length scale in the problem, we can actually simplify F even further, writing

$$F \sim \delta(a) - \frac{b}{a} \delta_R(a+b), \quad (\text{A.19})$$

where δ_R denotes an R -regulated delta function with width $1/R$ and height R . Physically, F takes this form because it receives two contributions, corresponding to exact momentum conservation and deviations from momentum conservation, set by the scale $1/R$.

Also, note that our results are parametrically correct even if the Higgs profile differs from the square bump function form which we have assumed. Looking at other forms for B , we find it is only really necessary that $B \approx 1$ in the region $x_3 \in [-R, R]$.

Next, let us integrate over phase space and compute the total probability of decay using this amplitude.

A.3 Probability

The decay rate for a $1 \rightarrow 2$ process is given by

$$\Gamma \sim \frac{1}{E} \int \frac{d^3k d^3q}{k_0 q_0} |\mathcal{M}|^2. \quad (\text{A.20})$$

In translationally invariant theories, \mathcal{M} is proportional to a 4-momentum conserving delta function, so $|\mathcal{M}|^2$ is a product of squares of delta functions. While naively this introduces a divergent $\delta(0)$ term, we normally divide Γ by the volume of spacetime, hence removing these factors. However, in our case, F has a component that exactly conserves 4-momentum and a component that violates momentum in the x_3 direction by an amount $1/R$. From basic kinematics, we know that the contribution to Γ from the 4-momentum conserving piece will not contribute, since a massless particle cannot decay into two massive particles in free space. For this reason, we will only need to compute the momentum violating contribution to $|F|^2$. Since we are not dividing by the size of the x_3 direction, we are actually computing the decay rate integrated over all of x_3 , i.e. the total probability of decay (see appendix B for details).

For an incoming momentum

$$p = (E, 0, 0, E), \quad (\text{A.21})$$

the (dimensionless) decay probability in our toy model is

$$P_{\phi \rightarrow \chi\chi} \sim \frac{g^2}{E} \int \frac{d^3k}{k_0} \frac{d^3q}{q_0} \delta(k_0 + q_0 - E) \delta(k_1 + q_1) \delta(k_2 + q_2) |F(k_3, q_3, E)|^2, \quad (\text{A.22})$$

where the energies are

$$k_0 = \sqrt{\vec{k}^2 + m_0^2}, \quad (\text{A.23})$$

$$q_0 = \sqrt{\vec{q}^2 + m_0^2}. \quad (\text{A.24})$$

It is trivial to integrate over the transverse q momenta, after which we parameterize the two transverse k momenta in polar coordinates as

$$(k_1, k_2) = k_\perp (\cos \theta, \sin \theta) \quad (\text{A.25})$$

$$d^3k = k_\perp dk_\perp d\theta dk_3. \quad (\text{A.26})$$

From the energy conservation delta function we obtain the useful expressions

$$k_0 = \frac{1}{2E} (E^2 + k_3^2 - q_3^2), \quad (\text{A.27})$$

$$q_0 = \frac{1}{2E} (E^2 + q_3^2 - k_3^2). \quad (\text{A.28})$$

Applying these formulae, the delta function becomes

$$\delta(k_0 + q_0 - E) = \frac{1}{k_\perp} \frac{k_0 q_0}{E} \times \delta(k_\perp - K_\perp), \quad (\text{A.29})$$

where K_\perp is defined as

$$K_\perp^2 = \frac{1}{4E^2} (k_3 + q_3 - E)(k_3 - q_3 - E)(k_3 + q_3 + E)(k_3 - q_3 + E) - m^2. \quad (\text{A.30})$$

The factor multiplying the delta function cancels with most of the integral, eliminating all k_\perp dependence except for the delta function! Note that for a completely 4-momentum conserving interaction, $k_3 + q_3 = E$, and so $K_\perp^2 = -m^2 < 0$ and the k_\perp integral yields zero, corresponding to the fact that a massless particle cannot decay into two massive particles if 4-momentum is conserved. Since we lack momentum conservation in the 3-direction, $K_\perp^2 > 0$ and the k_\perp integral instead gives unity. Consequently, we obtain the probability of a ϕ particle to decay into two χ particles

$$P_{\phi \rightarrow \chi\chi} \sim \frac{g^2}{E^2} \int dk_3 dq_3 |F(k_3, q_3, E)|^2. \quad (\text{A.31})$$

The domain of integration is the compact region

$$\sqrt{k_3^2 + m_0^2} + \sqrt{q_3^2 + m_0^2} \leq E, \quad (\text{A.32})$$

as derived from energy conservation. This inequality is saturated when k_\perp is zero. Note that we have made no approximations in evaluating this phase space integral, so the result, written in terms of F , is correct up to numerical coefficients.

A.4 From Toy Scalars to Gauge Bosons

Given $P_{\phi \rightarrow \chi\chi}$, it is straightforward to obtain $P_{A \rightarrow WW}$, the probability of a photon to decay to two W 's. The only parametric difference in the two calculations is that in the gauge theory calculation, g becomes a dimensionless coupling and the three gauge boson interaction has an extra factor of momentum due to the derivative coupling. This introduces an extra factor of E^2 into the decay probability. Moreover, since the outgoing W 's can be any of N gauge bosons, we also get a factor of N , yielding

$$P_{A \rightarrow WW} \sim \lambda \int dk_3 dq_3 |F(k_3, q_3, E)|^2. \quad (\text{A.33})$$

Before we plug in for F , let us pause to note a high energy subtlety which was mentioned earlier in terms of soft collinear divergences. Naively, we would be tempted to simply set $b = -a$ in F because of the delta function of $a + b$. However, since b is only fixed to be equal $-a$ up to a $1/R$ width, this replacement is only valid if b is of order $1/R$ or more, i.e. if

$$m_0^2 \left(\frac{1}{k_3} + \frac{1}{q_3} \right) > \frac{1}{R}, \quad (\text{A.34})$$

which is precisely the soft collinear bound derived in Eq. (3.2). As we will show, the energy scaling of the decay probability is quite different above and below this bound.

First, let us consider the $E < m_0^2 R$ regime. Setting $b = -a$ and integrating $|F|^2$ over k_3 and q_3 (ignoring contributions from the 4-momentum conserving delta function), we find that

$$P_{A \rightarrow WW} \sim \lambda \int dk_3 dq_3 |\delta_R(a+b)|^2 \quad (\text{A.35})$$

$$\sim \lambda \delta_R(0) \int dk_3, \quad (\text{A.36})$$

$$\sim \lambda E R, \quad (E < m_0^2 R) \quad (\text{A.37})$$

which is our final answer for the probability of a photon to decay to two W bosons at energies $E < m_0^2 R$.

Next, let us determine the probability of decay in the regime $E > m_0^2 R$, which we will divide into two phenomenologically distinct regions of phase space. First, consider the regime where k_3 is less than $m_0^2 R$ but q_3 is large enough that their sum, E , exceeds $m_0^2 R$ (obviously our result is symmetric under $k_3 \leftrightarrow q_3$). We will denote this as the “Bremsstrahlung” regime because one of the outgoing particles is much softer than the other. Since Eq. (A.34) obtains in this limit, the entire discussion of the previous paragraphs applies except that the k_3 integral is bounded by $k_3 < m_0^2 R$, and so

$$P_{A \rightarrow WW} \sim \lambda m_0^2 R^2. \quad (E > m_0^2 R, \text{ Bremsstrahlung}) \quad (\text{A.38})$$

If we are instead interested in a “symmetric decay,” defined by $k_3, q_3 > m_0^2 R$, then it becomes necessary to include the b/a in our expression for F , and the answer becomes

$$P_{A \rightarrow WW} \sim \frac{\lambda m_0^4 R^3}{E}. \quad (E > m_0^2 R, \text{ symmetric decay}) \quad (\text{A.39})$$

Note the crucial difference between these two regions of phase space – the first corresponds to decay products of energy $m_0^2 R$ and $E - m_0^2 R$ while the second corresponds to two decay products with energies roughly of order E .

A.5 Higher Order Effects and Approximations

Thus far we have only taken into account the role of three gauge boson interactions mediating binary decays in the plasma puddle atmosphere. However, it is fair to ask whether these contribution necessarily dominate over the four gauge boson interactions, which mediate trinary decays. In this section compare the relative sizes of $P_{A \rightarrow WWW}$ and $P_{A \rightarrow WW \rightarrow WWW}$, and argue that the former contribution is subdominant.

Repeating our procedure from section A.2, we find the contribution to the decay amplitude from the four gauge boson interaction,

$$\mathcal{M}(k, q, r, p) = g^2 \times \int d^4 x e^{i(\tilde{k}(x) + \tilde{q}(x) + \tilde{r}(x) - p)x}. \quad (\text{A.40})$$

Again, this amplitude simplifies to the form $\delta_R(a + b)$, where this time

$$a = k_3 + q_3 + r_3 - p_3, \quad (\text{A.41})$$

$$b = m_0^2 \left(\frac{1}{k_3} + \frac{1}{q_3} + \frac{1}{r_3} \right). \quad (\text{A.42})$$

Summing over N^2 possible final states, we find that

$$P_{A \rightarrow WWW} \sim \lambda^2 ER. \quad (\text{A.43})$$

Despite multiple phase space integrals, the form of this answer is expected because $|F|^2 \sim |\delta_R(a + b)|^2$ yields precisely one factor of R , and factors of E make up the rest of the expression.

Comparing, we see that the probability of one trinary decay $P_{A \rightarrow WWW}$ is much less than that of two consecutive binary decays

$$P_{A \rightarrow WW \rightarrow WWW} \sim (\lambda ER)^2, \quad (\text{A.44})$$

so it is the latter that dominates the $1 \rightarrow 3$ decay. Thus, the dominant contribution to the decay is binary and our expression from section 3.2 is valid. Similarly, loop effects are not large even though the $1 \rightarrow n$ rate is large at high energies, because only initial and final states involve propagators that are almost on-shell.

B. Rates and Cross Sections

Normally, we are interested quantities such as the cross section and decay rate. However, in our setup the “decay rate” is position dependent, and so to avoid this complication we simply compute the total probability that the incident particle will decay. This requires a slight revision of the standard formulas for converting S-matrix elements into physical observables.

Let us begin with Weinberg’s [13] formula Eq. 3.4.9. Putting the universe in a box of size L and in a time interval of size T , we have that

$$dP(\alpha \rightarrow \beta) \sim \frac{1}{L^3} |S_{\beta\alpha}|^2 d\beta. \quad (\text{B.1})$$

In a 4-momentum conserving theory, the S-matrix element is defined as

$$S_{\beta\alpha} \sim i\delta^{0123}(p_\beta - p_\alpha) M_{\beta\alpha}, \quad (\text{B.2})$$

where we identify $\delta(0) \sim L$. However, in our case momentum is violated in the x_3 direction, so we have that

$$S_{\beta\alpha} \sim i\delta^{012}(p_\beta - p_\alpha) F(p_{\beta 3} - p_{\alpha 3}). \quad (\text{B.3})$$

Thus the cancellation of volume factors will not be complete. Instead

$$dP(\alpha \rightarrow \beta) \sim \frac{T}{L} |M_{\beta\alpha}|^2 \delta^{012}(p_\beta - p_\alpha) |F(p_{\beta 3} - p_{\alpha 3})|^2 d\beta. \quad (\text{B.4})$$

Normally we would divide by T and take the $L \rightarrow \infty$ limit, but in our case this sends the probability to zero. This is just the statement that a particle in infinite volume will, on average, take an infinite time to hit the domain wall. Similarly, given a finite volume, if $T \rightarrow \infty$ then probability will blow up.

Thus we should take $T, L \rightarrow \infty$ with T/L held constant. It seems that this leads to an arbitrary factor, but physically this factor should be one, since it just corresponds to the number of times the particle crosses the domain wall.

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